

# Towards a Non-Perturbative Calculation of DIS Wilson Coefficients\*

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We verify the operator product expansion (OPE) of deep inelastic scattering (DIS) on the lattice and present first results of a non-perturbative calculation of the Wilson coefficients.

## 1. INTRODUCTION

The calculation of power corrections in DIS requires not only the evaluation of the matrix elements of higher-twist operators, but also the computation of the Wilson coefficients beyond perturbation theory [1]. In [2] we saw already that the scale dependence of the matrix elements of the twist-2 operators matches the scale dependence of the perturbatively calculated Wilson coefficients at most at large scales, so that we may expect large non-perturbative effects. A lattice calculation of the Wilson coefficients would also save us to renormalize the lattice operators.

We consider DIS off a quark target. This is sufficient for the calculation of the ‘leading’ Wilson coefficients. The OPE reads

$$W_{\mu\nu}(p, q) \equiv \langle p | J_\mu(q) J_\nu(-q) | p \rangle + \text{seagull} \\ = \sum_n C_{n,\mu\nu}(\Lambda^{-1} q) O_n(\Lambda^{-1} p), \quad (1)$$

where  $|p\rangle$  is an off-shell quark state of momentum  $p$  which is struck by a photon of momentum  $q$ , and  $\Lambda$  is a scale parameter. The  $C_{n,\mu\nu}(\Lambda^{-1} q)$  are the Wilson coefficients, and  $O_n(\Lambda^{-1} p)$  are the forward matrix elements of the operators

$$\mathcal{O}_{\mu_1 \dots \mu_n} = \bar{\psi} \Gamma_{\mu_1} D_{\mu_2} \dots D_{\mu_n} \psi, \quad (2)$$

where  $\Gamma$  is a matrix in Dirac space. In the following we will distinguish between bare and renor-

malized quantities. In the former  $\Lambda = a^{-1}$ , where  $a$  is the lattice spacing. In the latter  $\Lambda = \mu$ , where  $\mu$  is the renormalization scale.

The Wilson coefficients are independent of the target. Let us denote the nucleon quantities by the superscript  $N$ . So if we multiply the nucleon matrix elements of the operators (2) by the corresponding Wilson coefficients, we get the nucleon structure functions:

$$W_{\mu\nu}^{(N)}(p, q) = \sum_n C_{n,\mu\nu}(\Lambda^{-1} q) O_n^{(N)}(\Lambda^{-1} p). \quad (3)$$

The r.h.s. of (3), being independent of  $\Lambda$ , can be written in different ways:

$$C_{n,\mu\nu}(a q) O_n^{(N)}(a p) \\ = C_{n,\mu\nu}(\mu^{-1} q) O_n^{(N)}(\mu^{-1} p) \\ = C_{n,\mu\nu}(\mu^{-1} q) Z_n(\mu a) O_n^{(N)}(a p), \quad (4)$$

where  $Z_n(\mu a)$  is the renormalization constant of the lattice operator [2,3]. As a result, the renormalized and bare Wilson coefficients are related by

$$C_{n,\mu\nu}(\mu^{-1} q) = Z_n^{-1}(\mu a) C_{n,\mu\nu}(a q). \quad (5)$$

The nucleon structure functions are then most conveniently computed from the product of bare Wilson coefficients and lattice nucleon matrix elements. A further advantage of this approach is that operator mixing with higher-twist operators is automatically included.

\*Talk presented by D. Petters at Lattice 1998.

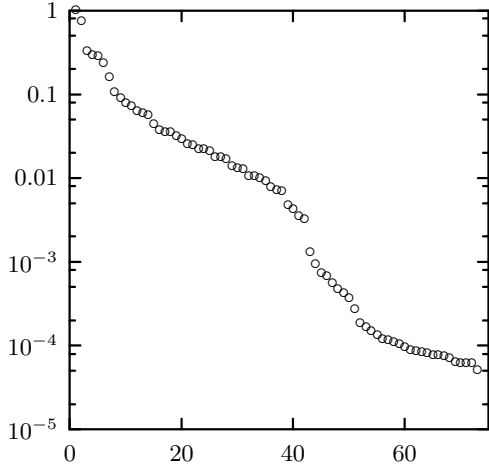


Figure 1. The eigenvalues  $w_n$  against  $n$ , normalized so that the largest value is one.

The plan is now to compute the tensor  $W_{\mu\nu}$  and the operator matrix elements  $O_n$  for a quark in the Landau gauge, and to extract the Wilson coefficients from this information.

## 2. THE METHOD

We consider a system of quark momenta  $p_m$ ,  $1 \leq m \leq M$  with the photon momentum being fixed. The Wilson coefficients are independent of  $p_m$ , so the problem is to solve the  $N \times M$  system of equations:

$$\begin{pmatrix} O_1^{p_1} & \dots & O_N^{p_1} \\ \vdots & & \vdots \\ O_1^{p_M} & \dots & O_N^{p_M} \end{pmatrix} \begin{pmatrix} C_1 \\ \vdots \\ C_N \end{pmatrix} = \begin{pmatrix} W^{p_1} \\ \vdots \\ W^{p_M} \end{pmatrix}, \quad (6)$$

where  $O_n^{p_m}$  are the ensemble averaged (amputated) matrix elements of the operators (2) between quark states of momentum  $p_m$ ,  $W^{p_m}$  are the corresponding elements of the hadronic tensor, and the  $C_n$  are the Wilson coefficients we are looking for. The label  $n$  runs over the various operators. Note that Lorentz and Dirac indices have been omitted. We shall take all operators with up to three covariant derivatives into account. Since we do not symmetrize over the indices  $\mu_1, \mu_2, \dots$ , this includes higher-twist operators as well.

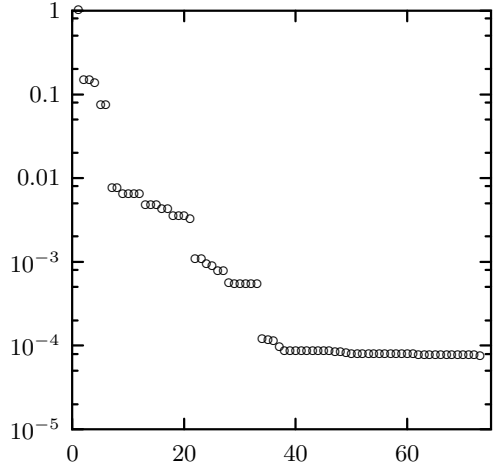


Figure 2. The residual error  $R^2$  against  $n$ .

We write (6) as

$$OC = W. \quad (7)$$

To compute  $C$ , we apply a singular value decomposition, which is the standard method of solving overdetermined equations [4]. We write

$$O = UwV^T, \quad (8)$$

where  $U$  is a column-orthonormal  $M \times N$  matrix,  $w$  is a diagonal  $N \times N$  matrix,  $w = \text{diag}(w_n)$ , with positive real eigenvalues  $w_n$ ,  $1 \leq n \leq N$  arranged in descending order, and  $V^T$  is the transpose of a column-orthonormal  $N \times N$  matrix. The solution  $C$  is obtained by applying

$$V \text{diag}(1/w_n) U^T \quad (9)$$

to  $W$ . To find the vector  $C$  of smallest length, we replace  $1/w_n$  by zero if  $w_n$  is dominated by noise.

## 3. RESULTS

The calculations are done with standard Wilson fermions on  $24^3 48$  lattices at  $\beta = 6.2$  and  $\kappa = 0.1489$ . So far we have looked at two configurations. We employed 71 different quark momenta. For the photon momentum we took

$$q^2 = (\pi/2a)^2 = 17.4 \text{ GeV}^2. \quad (10)$$

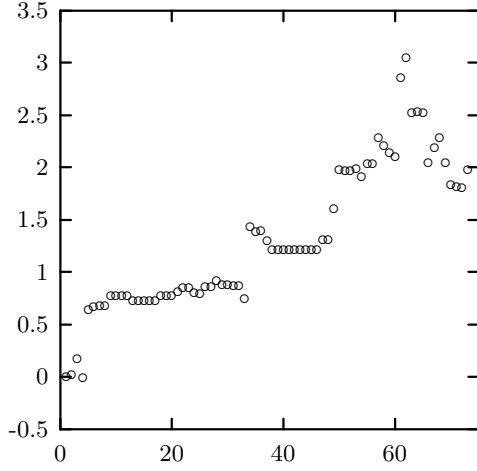


Figure 3. The Wilson coefficient  $c_x(aq)$  of the operator  $X = \bar{\psi}\gamma_{\{i}D_{j\}}\psi$  with  $i \neq j$  against  $n$ .

In Fig. 1 we show the eigenvalues  $w_n$  of  $w$ . We see a sharp decrease between  $n \approx 40$  and  $50$  by approximately a factor of 30, and the  $w_n$  for larger  $n$  are probably dominated by noise. In Fig. 2 we have plotted the residual error  $R^2 = |W - OC|^2/|W|^2$  that results from setting  $1/w_m = 0, \forall m \geq n$ , as a function of  $n$ . The error drops sharply until  $n$  reaches  $\approx 40$  and then stays constant, saying that if one makes a fit with too few parameters one does not get a good fit, and if one fits with too many one ‘fits to the noise’. So we must truncate the system at  $n \approx 40 - 50$ .

The commonly used Wilson coefficients (bare of all kinematical factors) are obtained by

$$c_n(aq) = C_{n,\mu\nu}(aq)/C_{n,\mu\nu}^{\text{tree}}(aq), \quad (11)$$

where  $C_{n,\mu\nu}^{\text{tree}}(aq)$  is the tree value. In Fig. 3 we show the Wilson coefficient  $c_x(aq)$  [5] of the operator  $X = \bar{\psi}\gamma_{\{i}D_{j\}}\psi$  with  $i \neq j$ , again for various degrees of truncation. We find a plateau in the region where the eigenvalues  $w_n$  drop to zero, so that it does not matter where exactly we truncate the system. We have chosen  $n = 42$ . Beyond the plateau the errors on  $w_n$  get large.

As a first step we have verified the OPE. In Fig. 4 we show the Wilson coefficients  $c_n(aq)$  of the lower dimensional operators. We compare our results with the tree-level predictions. We

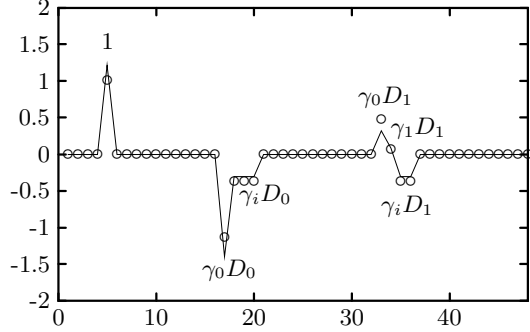


Figure 4. The Wilson coefficients  $c_n(aq)$  of the first 48 operators. The open circles are our Monte-Carlo results. The solid line is the tree-level result. The horizontal axis labels the operators, including the 16 different  $\Gamma$  matrices.

find that the expected structure is very well reproduced. In particular, the Wilson coefficients which should be zero by symmetry or other arguments are actually found to be zero.

As a first application of our method we have determined the Wilson coefficient of the operator  $X$  already shown in Fig. 3. This operator gives the first (non-trivial) moment of the unpolarized structure function [5],  $\langle x \rangle$ . We find (at  $q^2 = 17.4 \text{ GeV}^2$ , cf. (10))  $c_x(aq) \approx 1.2$ . If we divide this number by the corresponding renormalization constant,  $Z_x^{\overline{MS}}(\mu a)$ , as given in [2], we obtain

$$c_x^{\overline{MS}}(\mu^{-1}q)|_{\mu^2=q^2} \approx 1.2. \quad (12)$$

This is to be compared with the 2-loop result of  $c_x^{\overline{MS}}(\mu^2 = q^2) = 1.01$ .

## REFERENCES

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